PG – 854

I Semester M.Sc. Degree Examination, January 2017
(R.N.S.) (2011 Onwards)
MATHEMATICS
M 101 : Algebra – I

Time : 3 Hours   Max. Marks : 80

***Instructions**: 1) Answer **any five** questions choosing atleast **two** from **each** Part.*
*2) **All** questions carry **equal** marks.*

PART – A

1. a) State and prove Cayley's theorem for finite groups. **5**

   b) Define an isomorphism of groups with usual notations prove that $G/N \approx (G/K)/(N/K)$. **6**

   c) Determine : (i) $Aut(K_4)$ (ii) $Inn(K_4)$, where K_4 is Klein four group. Hence illustrate that the automorphism group of an abelian group need not be abelian. **5**

2. a) State and prove the Cauchy-Frobenius lemma. **6**

   b) Verify the class equation for symmetric group S_3. **5**

   c) Prove that every group of order p^2, for a prime p, is abelian. **5**

3. a) State and prove Sylow's first theorem. **10**

   b) Let $o(G) = pq$, where p and q are primes with $p < q$ and $q \not\equiv 1 \pmod{p}$. Then show that G is cyclic and abelian. **6**

4. a) Show that any group of order 30 is not simple. **4**

   b) State and prove Jordan-Holder theorem for solvable groups. **8**

   c) Prove that a subgroup of a solvable group is solvable. **4**

P.T.O.

PART – B

5. a) Define a division ring. Prove that Q(R), the ring of quoternians is a division ring. **5**

b) Let R be a commutative ring with unity whose ideals are {0} and R only. Prove that R is a field. **5**

c) If U, V are ideals of a ring R, let UV be the set of all elements that can be written as a finite sums of elements of the form uv, where $u \in U$, $v \in V$. Prove that UV is an ideal of R. **6**

6. a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field. **6**

b) Let P_1 and P_2 be prime ideals in a commutative ring R. Show that $P_1 \cap P_2$ is prime implies $P_1 \subset P_2$ or $P_2 \subset P_1$. **5**

c) Prove that any two isomorphic integral domains have isomorphic quotient fields. **5**

7. a) Define Euclidean ring. Prove that Z[i] is a Euclidean ring. **6**

b) Let R be a ring. Prove that any two elements a and b in R have a g.c.d. d. Moreover $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$. **5**

c) If p is a prime number of the form 4n + 1, then show that $x^2 \equiv -1 \pmod p$ has a solution. **5**

8. a) Prove that F[x] is a principal ideal ring, where F is a field. **5**

b) Define : (i) primitive polynomial (ii) content of a polynomial. Prove that the product of two primitive polynomials is primitive. Hence deduce that c(fg) = c(f) c(g). **7**

c) State and prove Gauss lemma for the primitive polynomials. **4**